# **Problem 2:**

1. To derive the motion tracking equation from fundamental principles, we start by considering the motion of an object between two consecutive frames in a video sequence. Let's denote the position of a point in the first frame as (x,y)(*x*,*y*), and its position in the second frame as (x+Δx,y+Δy)(*x*+Δ*x*,*y*+Δ*y*), where (Δx,Δy)(Δ*x*,Δ*y*) represents the motion of the point.

The motion tracking equation can be derived by considering the optical flow constraint, which states that the intensity of a point in the first frame should be equal to the intensity of the corresponding point in the second frame. Mathematically, this can be expressed as:

*I*(*x*,*y*,*t*)=*I*(*x*+Δ*x*,*y*+Δ*y*,*t*+Δ*t*)

where I(x,y,t)*I*(*x*,*y*,*t*) is the intensity of the point at position (x,y)(*x*,*y*) in the first frame at time t*t*, and I(x+Δx,y+Δy,t+Δt)*I*(*x*+Δ*x*,*y*+Δ*y*,*t*+Δ*t*) is the intensity of the corresponding point in the second frame at time t+Δt*t*+Δ*t*.

A close-up of math equations

Description automatically generatedExpanding this equation using a Taylor series approximation and keeping only the first-order terms, we get:

This equation represents the relationship between the spatial and temporal gradients of intensity and the motion of a point in an image sequence.

To compute the motion function estimates between two consecutive frames, you can use techniques like Lucas-Kanade algorithm, Horn-Schunck algorithm, or optical flow methods.

1. To perform Lucas-Kanade algorithm for motion tracking when the motion is known to be affine, we can adapt the algorithm to estimate the affine parameters a1,b1,c1,a2,b2,c2*a*1​,*b*1​,*c*1​,*a*2​,*b*2​,*c*2​ representing the motion field.

The Lucas-Kanade algorithm estimates the optical flow for each pixel by solving a linear system of equations in a local neighborhood around the pixel. When the motion is known to be affine, the optical flow equation becomes:

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where Ix*Ix*​, Iy*Iy*​, and It*It*​ are the spatial and temporal gradients of intensity at the pixel location. We can rewrite this equation as:A black and white image of a rectangular object with letters and numbers

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where *A* is the matrix of spatial gradients and It*It*​ is the temporal gradient.

To solve for the affine parameters a1,a2,b1,b2,c1,c2*a*1​,*a*2​,*b*1​,*b*2​,*c*1​,*c*2​, we can use methods like least squares estimation or singular value decomposition (SVD). Once the parameters are estimated, they can be used to represent the motion field between the two frames.